

## A.2 Block-diagonal Hessian

This section contains models with grouped or nested random effects

### A.2.1 Nonlinear mixed models; a NLME comparison

**Model description** The orange tree growth data was used by Pinheiro & Bates (2000, Ch.8.2) to illustrate how a logistic growth curve model with random effects can be fit with the S-Plus function `nlme`. The data contain measurements made at seven occasions for each of five orange trees:

$t_{ij}$  Time point when the  $j$ th measurement was made on tree  $i$   
 $y_{ij}$  Trunk circumference of tree  $i$  when measured at time point  $t_{ij}$

The following logistic model is used:

$$y_{ij} = \frac{\phi_1 + u_i}{1 + \exp[-(t_{ij} - \phi_2)/\phi_3]} + \varepsilon_{ij},$$

where  $(\phi_1, \phi_2, \phi_3)$  are hyper-parameters, and  $u_i \sim N(0, \sigma_u^2)$  is a random effect, and  $\varepsilon_{ij} \sim N(0, \sigma^2)$  is the residual noise term.

**Results** Parameter estimates are shown in the following table.

	$\phi_1$	$\phi_2$	$\phi_3$	$\sigma$	$\sigma_u$
ADMB-RE	192.1	727.9	348.1	7.843	31.65
Std. dev.	15.658	35.249	27.08	1.013	10.26
<code>nlme</code>	191.0	722.6	344.2	7.846	31.48

The difference between the estimates obtained with ADMB-RE and `nlme` is small. The difference is caused by the fact that the two approaches use different approximations to the likelihood function. (ADMB-RE uses the Laplace approximation, and for `nlme` the reader is referred to (Pinheiro & Bates 2000, Ch. 7).)

The computation time for ADMB was 0.58 seconds, while the computation time for `nlme` (running under S-Plus 6.1) was 1.6 seconds.

**Files** <http://otter-rsch.com/admbre/examples/orange/orange.html>

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